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Exactly solving the general non-degenerate multimode multiphoton Jaynes–Cummings model with field nonlinearity

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Abstract
For the general non-degenerate multimode multiphoton Jaynes–Cummings (JC) model, including any forms of intensity-dependent coupling, field-dependent detuning and field nonlinearity, we obtain its energy eigenvalues and eigenstates via the supersymmetric unitary transformation (SUT) method. In addition, its pseudo-invariant eigen-operator (PIEO) is also found, which directly leads to the energy-level gap.

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1. Introduction

The interaction of atoms with an electromagnetic field is attracting much attention from many researchers in the field of quantum optics [1–3]. The simplest physical situation can be described by the well-known Jaynes–Cummings (JC) model [4], where the interaction of a single two-level atom with one cavity field mode near resonance is studied under the condition of the rotating-wave approximation and exhibits many interesting quantum effects. Over many years, the extensions of the basic JC model have included intensity-dependent coupling constants [5], two-photon or multi-photon transitions [6], two cavity modes [7] for three-level atoms as well as more complex systems [8] and so on. In the investigations of these models, it is a basic task to get their energy spectra and eigenstates. However, these systems, which can be solved both exactly and analytically, are very limited because many differential equations are hardly solvable, so researchers have been trying to find other ways to overcome this obstacle. In [9], it was proposed that the JC model can be solved by a supersymmetric unitary transformation (SUT). Subsequently some authors [10] further applied this method

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to calculate the eigenvalues and eigenstates for some other JC models. Xu *et al* [11] used a dynamical algebraic method to obtain the solutions for some modified JC Hamiltonian. In very recent works [12], Fan *et al* have reported invariant eigen-operator (IEO) theory and further extended the pseudo-invariant eigen-operator (PIEO) method. It is worth mentioning that the PIEO method may more simply obtain the energy levels of some generalized JC models [13].

In this paper, based on the above ideas, we further use the SUT method and the PIEO method to study the non-degenerate general JC model, which consists of an effective two-level atom with a bare transition frequency ω_0 and quantized multimodels of a lossless cavity with different frequencies ω_i, respectively. In the rotating-wave approximation, the Hamiltonian of this system is given by (setting $\hbar = 1$)

$$H_m = \sum_{i=1}^{m} \omega_i N_i + \frac{1}{2}\omega_0 G(\{N_i\})\sigma_z + R(\{N_i\}) + \chi[A_m f(\{N_i\})\sigma_+ + f(\{N_i\})A_m^\dagger \sigma_-], \tag{1}$$

which includes any form of intensity-dependent coupling, field-dependent detuning and field nonlinearity, where $A_m^\dagger = \prod_{j=1}^m a_j^\dagger$ (for the degenerate case $A_m^\dagger = a^{\dagger m}$) are m-photon transition operators, and σ_z is the two-level (denoted by $|-\rangle$ and $|+\rangle$) atomic inversion operator, defined as

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad \sigma_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \qquad \sigma_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \tag{2}$$

obeying

$$[\sigma_z, \sigma_\pm] = \pm 2\sigma_\pm, \qquad [\sigma_+, \sigma_-] = \sigma_z. \tag{3}$$

The ith field mode ($i = 1, 2, \ldots, m$) is characterized by bosonic creation and annihilation operators $a_i^\dagger$ and a_i. Here $G(\{N_i\}) \equiv G(N_1, N_2, \ldots, N_m)$, $R(\{N_i\}) \equiv R(N_1, N_2, \ldots, N_m)$ and $f(\{N_i\}) \equiv f(N_1, N_2, \ldots, N_m)$ are Hermitian operators and they are any reasonable functions of the photon number operators $N_i = a_i^\dagger a_i$. $G(\{N_i\})$ denotes the field-dependent detuning, $R(\{N_i\})$ represents the field nonlinearity item, and $\chi f(\{N_i\})$ represents the intensity-dependent atom–field coupling. Such a model of the atom–cavity interaction may have a realization in an experiment of a continuous-wave maser operating on a multiphoton transition between Rydberg levels; it is expected that the maser oscillates with about one atom and a few tens of microwave photons at any time in the cavity.

This model in equation (1) is a fairly general form. For instance, we consider the $m = 2$ case, namely, the two-mode two-photon Jaynes–Cummings (TTJC) model. If $f = G = 1$ and $R = 0$, it reduces to the conventional non-degenerate TTJC model [14]. If $R = 0$, it recovers as the intensity-dependent non-degenerate TTJC model [15]. It reduces to the model used by Gao *et al* [16] when $R = 0$, $G = 1$ and $f = \sqrt{a_1^\dagger a_1 a_2^\dagger a_2}$. As $f = G = 1$, $R = \frac{1}{4}\varepsilon_1 \left(a_1^\dagger a_1\right)^2 + \frac{1}{4}\varepsilon_2 \left(a_2^\dagger a_2\right)^2 + \varepsilon a_1^\dagger a_1 a_2^\dagger a_2$, we recover the non-degenerate TTJC model inside a high-Q cavity field with a Kerr-like medium [17]. In section 2, our main task is to define the supersymmetric generators to rewrite the Hamiltonian in equation (1) and diagonalize the JC model to obtain the energy eigenvalues and eigenstates. In section 3, based on the expression via supersymmetric generators, we go on searching for its so-called PIEO, which may directly lead to its energy-level gap. Section 4 concludes our paper with a summary and some discussion.

2. Supersymmetric generators and diagonalization for the general JC Hamiltonian

In order to construct the supersymmetric unitary transformation operator, we first denote the supersymmetric generators as

$$Q \equiv f(N_i)A_m^\dagger \sigma_- = \begin{pmatrix} 0 & 0 \\ f(N_i)A_m^\dagger & 0 \end{pmatrix} \tag{4}$$

$$Q^\dagger \equiv A_m f(N_i)\sigma_+ = \begin{pmatrix} 0 & A_m f(N_i) \\ 0 & 0 \end{pmatrix} \tag{5}$$

$$\begin{aligned} N' &\equiv A_m f^2(N_i)A_m^\dagger \sigma_{++} + f^2(N_i)A_m^\dagger A_m \sigma_{--} \\ &= \begin{pmatrix} A_m f^2(N_i)A_m^\dagger & 0 \\ 0 & f^2(N_i)A_m^\dagger A_m \end{pmatrix}, \end{aligned} \tag{6}$$

where

$$\sigma_{++} \equiv \sigma_+\sigma_- = \tfrac{1}{2}(1+\sigma_z), \qquad \sigma_{--} \equiv \sigma_-\sigma_+ = \tfrac{1}{2}(1-\sigma_z),$$

so that $N, Q, Q^\dagger$ constitute the supersymmetric generators and satisfy the commutation and anticommutation relations

$$\begin{aligned} &Q^2 = Q^{\dagger 2} = 0, \qquad [Q^\dagger, Q] = N'\sigma_z, \qquad (Q^\dagger - Q)^2 = -N', \\ &[N', Q] = [N', Q^\dagger] = 0, \qquad [Q, \sigma_z] = 2Q, \qquad [Q^\dagger, \sigma_z] = -2Q^\dagger, \\ &\{Q, \sigma_z\} = \{Q^\dagger, \sigma_z\} = 0, \qquad \{Q, Q^\dagger\} = N'. \end{aligned} \tag{7}$$

In terms of the generators, we can rewrite the Hamiltonian equation (1) as

$$\begin{aligned} H_m = \sum_{i=1}^m \left(M_i - \frac{1}{2}\right)\omega_i - \frac{1}{2}\sum_{i=1}^m \omega_i\sigma_z + \frac{1}{2}\omega_0 G(\{M_i - \sigma_{++}\})\sigma_z \\ + R(\{M_i - \sigma_{++}\}) + \chi(Q + Q^\dagger), \end{aligned} \tag{8}$$

where $M_i = N_i + \sigma_{++}$ is a constant of motion and obeys the commutation relations

$$[M_i, N'] = [M_i, Q] = [M_i, Q^\dagger] = 0. \tag{9}$$

Using the property $\sigma_{++}^2 = \sigma_{++}$, it is easily obtained that

$$\begin{aligned} G(\{M_i - \sigma_{++}\}) &= \sum_{l=1}^\infty \frac{G^{(l)}(\{0\})}{l!}\left[M_i^l + (M_i - 1)^l\sigma_{++} - M_i^l\sigma_{++}\right] \\ &= G(\{M_i\}) + [G(\{M_i - 1\}) - G(\{M_i\})]\sigma_{++} \\ &= G_+(\{M_i\}) + G_-(\{M_i\})\sigma_z, \end{aligned} \tag{10}$$

in which

$$G_+(\{M_i\}) = \tfrac{1}{2}[G(\{M_i - 1\}) + G(\{M_i\})], \tag{11}$$

$$G_-(\{M_i\}) = \tfrac{1}{2}[G(\{M_i - 1\}) - G(\{M_i\})]. \tag{12}$$

Similarly,

$$R(\{M_i - \sigma_{++}\}) = R_+(\{M_i\}) + R_-(\{M_i\})\sigma_z, \tag{13}$$

$$R_+(\{M_i\}) = \tfrac{1}{2}[R(\{M_i - 1\}) + R(\{M_i\})], \tag{14}$$

$$R_-(\{M_i\}) = \tfrac{1}{2}[R(\{M_i - 1\}) - R(\{M_i\})]. \tag{15}$$

From equations (10)–(15), equation (8) may be re-expressed as

$$H_m = H_{m0} + \tfrac{1}{2}\Delta(\{M_i\})\sigma_z + \chi(Q + Q^\dagger), \tag{16}$$

where

$$H_{m0} = \sum_{i=1}^{m} \left(M_i - \frac{1}{2}\right)\omega_i + \frac{1}{2}G_-(\{M_i\})\omega_0 + R_+(\{M_i\}), \tag{17}$$

$$\Delta(\{M_i\}) = 2R_-(\{M_i\}) + G_+(\{M_i\})\omega_0 - \sum_{j=1}^{m}\omega_j. \tag{18}$$

Now, with the help of the supersymmetric transformation generators mentioned above, we diagonalize the Hamiltonian in equation (16). The supersymmetric unitary transformation operator is defined as

$$T = \exp\left\{-\frac{\theta}{2}N'^{-1/2}(Q^\dagger - Q)\right\}, \tag{19}$$

where θ is a function of operators M_i to be determined later, and $N'^{1/2}$ is

$$N'^{-1/2} = \begin{pmatrix} \frac{1}{\sqrt{A_m f^2(\{N_i\})A_m^\dagger}} & 0 \\ 0 & \frac{1}{\sqrt{f^2(\{N_i\})A_m^\dagger A_m}} \end{pmatrix}. \tag{20}$$

Considering $a^\dagger f(aa^\dagger) = f(a^\dagger a)a^\dagger$, $af(a^\dagger a) = f(a^\dagger a + 1)a$, we easily get

$$[N'^{-1/2}, Q^\dagger] = [N'^{-1/2}, Q] = [N'^{-1/2}, M_i] = 0. \tag{21}$$

Therefore, from equations (7) and (21), equation (19) may be expanded to the following form:

$$T = \cos\frac{\theta}{2} - N'^{-1/2}(Q^\dagger - Q)\sin\frac{\theta}{2}, \tag{22}$$

so

$$T^{-1} = \cos\frac{\theta}{2} + N'^{-1/2}(Q^\dagger - Q)\sin\frac{\theta}{2} = T^\dagger. \tag{23}$$

Using equations (7), (22) and (23), it then follows that

$$T^{-1}H_{m0}T = H_{m0}, \tag{24}$$

$$T^{-1}(Q^\dagger + Q)T = (Q^\dagger + Q)\cos\theta + N'^{1/2}\sigma_z\sin\theta, \tag{25}$$

and

$$T^{-1}\sigma_z T = \sigma_z\cos\theta - N'^{-1/2}(Q^\dagger + Q)\sin\theta. \tag{26}$$

Based on the above equations, equation (16) can be easily solved by T^{-1},

$$\begin{aligned} H'_m &= T^{-1}H_m T \\ &= H_{m0} + \left[\chi\cos\theta - \tfrac{1}{2}\Delta(\{M_i\})N'^{-1/2}\sin\theta\right](Q + Q^\dagger) \\ &\quad + \left[\chi N'^{1/2}\sin\theta + \tfrac{1}{2}\Delta(M_i)\cos\theta\right]\sigma_z. \end{aligned} \tag{27}$$

Formally, if we annihilate the second term of equation (27) by letting

$$\frac{1}{N'^{1/2}}\tan\theta = \frac{2\chi}{\Delta(\{M_i\})}, \tag{28}$$

we can obtain the diagonalized Hamiltonian as follows:

$$H'_m = H_{m0} + \tfrac{1}{2}\sqrt{\Delta^2(\{M_i\}) + 4\chi^2 N'}\sigma_z. \tag{29}$$

Note that equation (28) should be understood in the sense of the eigenvalue equation for the operators M_i and N'. The corresponding eigenstates of H_m' are

$$\begin{aligned}|\Phi_1'\rangle &= |n_1\rangle \otimes |n_2\rangle \otimes \cdots \otimes |n_m\rangle \otimes |+\rangle \\ &\equiv \begin{pmatrix} |n_1, n_2, \ldots, n_m\rangle \\ 0 \end{pmatrix},\end{aligned} \tag{30}$$

$$\begin{aligned}|\Phi_2'\rangle &= |n_1+1\rangle \otimes |n_2+1\rangle \otimes \cdots \otimes |n_m+1\rangle \otimes |-\rangle \\ &\equiv \begin{pmatrix} 0 \\ |n_1+1, n_2+1, \ldots, n_m+1\rangle \end{pmatrix},\end{aligned} \tag{31}$$

and the eigen-equations of H_m' are given by

$$H_m'|\Phi_1'\rangle = [E_{m0}(\{n_i\}) + E_{m1}(\{n_i\})]|\Phi_1'\rangle, \tag{32}$$

$$H_m'|\Phi_2'\rangle = [E_{m0}(\{n_i\}) - E_{m1}(\{n_i\})]|\Phi_2'\rangle, \tag{33}$$

where

$$\begin{aligned}E_{m0}(\{n_i\}) &\equiv E_{m0}(n_1, n_2, \ldots, n_m) \\ &= \sum_{i=1}^{m}\left(n_i + \frac{1}{2}\right)\omega_i + \frac{1}{2}G_-(\{n_i+1\})\omega_0 + R_+(\{n_i+1\}),\end{aligned} \tag{34}$$

$$\begin{aligned}E_{m1}(\{n_i\}) &\equiv E_{m1}(n_1, n_2, \ldots, n_m) \\ &= \frac{1}{2}\sqrt{\Delta^2(\{n_i+1\}) + 4\chi^2 f^2(\{n_i+1\})\prod_{j=1}^{m}(n_j+1)}.\end{aligned} \tag{35}$$

Then the energy-level gap for this system is

$$\triangle E = 2E_{m1}(\{n_i\}) = \sqrt{\Delta^2(\{n_i+1\}) + 4\chi^2 f^2(\{n_i+1\})\prod_{j=1}^{m}(n_j+1)}. \tag{36}$$

From the above analysis, the corresponding eigenvalues and eigenstates of H_m in equation (1) are given by, respectively,

$$E_{m+} = E_{m0}(\{n_i\}) + E_{m1}(\{n_i\}), \tag{37}$$

$$|\Phi_1\rangle = T|\Phi_1'\rangle = \cos\left(\frac{\theta}{2}\right)|\Phi_1'\rangle + \sin\left(\frac{\theta}{2}\right)|\Phi_2'\rangle, \tag{38}$$

$$E_{m+} = E_{m0}(\{n_i\}) - E_{m1}(\{n_i\}), \tag{39}$$

$$|\Phi_2\rangle = T|\Phi_2'\rangle = \cos\left(\frac{\theta}{2}\right)|\Phi_2'\rangle - \sin\left(\frac{\theta}{2}\right)|\Phi_1'\rangle, \tag{40}$$

where

$$\cos\left(\frac{\theta}{2}\right) = \frac{1}{\sqrt{2}}\sqrt{1 + \frac{\Delta(\{n_i+1\})}{2E_{m1}(\{n_i\})}}, \tag{41}$$

$$\sin\left(\frac{\theta}{2}\right) = \frac{1}{\sqrt{2}}\sqrt{1 - \frac{\Delta(\{n_i+1\})}{2E_{m1}(\{n_i\})}}. \tag{42}$$

It should be pointed out that the states $\begin{pmatrix} 0 \\ |n_1,0,\ldots,0\rangle \end{pmatrix}, \begin{pmatrix} 0 \\ |0,n_2,\ldots,0\rangle \end{pmatrix}, \ldots, \begin{pmatrix} 0 \\ |0,0,\ldots,n_m\rangle \end{pmatrix}$, which are not included in the above discussion, are also eigenstates of H_m.

3. Finding the pseudo-invariant eigen-operator for the general JC Hamiltonian

To begin with, we briefly review and explain the so-called PIEO method [12]. Let us trace back the original idea of the Schrödinger quantization scheme, where the identification $\mathrm{i}\frac{d}{\mathrm{d}t} \leftrightarrow \hat{H}$ (Hamiltonian), so $\mathrm{i}\frac{d}{\mathrm{d}t}$ is named the Schrödinger operator in many references. Similarly, we have $\left(\mathrm{i}\frac{d}{\mathrm{d}t}\right)^n \leftrightarrow \hat{H}^n$; now we set up the n-order differential equation for an operator $\hat{O}_e$,

$$\left(\mathrm{i}\frac{d}{\mathrm{d}t}\right)^n \hat{O}_e = \lambda \hat{O}_e. \tag{43}$$

When $n = 1$, it looks like the equation $\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}\psi = \hat{H}\psi(\hbar = 1)$. Thus, equation (43) is called the n-order invariant eigen-operator equation with the n-order eigenvalue. Using the Heisenberg equation $\mathrm{i}\frac{d}{\mathrm{d}t}\hat{O}_e = [\hat{O}_e, \hat{H}]\,(\hbar = 1)$, equation (43) is rewritten as

$$\left(\mathrm{i}\frac{d}{\mathrm{d}t}\right)^n \hat{O}_e = [\ldots[[\hat{O}_e, \hat{H}], \hat{H}]\ldots, \hat{H}] = \lambda \hat{O}_e. \tag{44}$$

If such an $\hat{O}_e$ is found, $\sqrt[n]{\lambda}$ is called the energy-level gap. When for some Hamiltonian the n-fold commutator $[\ldots[[\hat{O}_e, \hat{H}], \hat{H}]\ldots, \hat{H}]$ in equation (44) is not proportional to $\hat{O}_e$, it seems that the next $(n + 1)$-fold commutator will not produce a constant multiplied by $\hat{O}_e$ either; then if there exists some state-vector space spanned by $|\phi\rangle_i$ in which the equation $[\ldots[[\hat{O}_e, \hat{H}], \hat{H}]\ldots, \hat{H}]|\phi\rangle_i = \lambda\hat{O}_e|\phi\rangle_i$ holds; in this limited space $\hat{O}_e$ is called the pseudo-invariant eigen-operator of $\hat{H}$. Usually, $|\phi\rangle_i$ is the eigenvector of conservative quantities of the dynamic system which commute with the Hamiltonian. Then, the equation

$$\left(\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}\right)^n \hat{O}_e|\phi\rangle_i = \lambda\hat{O}_e|\phi\rangle_i \tag{45}$$

may lead us to obtain some information of energy gap of the Hamiltonian.

Next, based on the above Hamiltonian in equation (16) described by the supersymmetric generators, we will search for its PIEO and derive its energy-level gap. We first assume that the PIEO of Hamiltonian in equation (16) possesses the form

$$\hat{O}_e = \alpha(Q^\dagger + Q) + \beta\sigma_z, \tag{46}$$

where α and β are undermined constants. Using the relations in equations (7) and (21), we calculate

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}\hat{O}_e = [\hat{O}_e, \hat{H}] = [\alpha\Delta(\{M_i\}) - \beta\chi](Q - Q^\dagger). \tag{47}$$

Further calculation shows

$$\left(\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}\right)^2 \hat{O}_e = [\alpha\Delta(\{M_i\}) - \beta\chi][\Delta(\{M_i\})(Q^\dagger + Q) - \chi N'\sigma_z]. \tag{48}$$

Comparing the right-hand side of equation (48) with equation (46), it is unlikely that $\hat{O}_e$ in equation (46) can satisfy the eigen-operator equation (44) for the $n = 2$ case. However, according to the PIEO theory, with the two sides of equation (48) acting on the eigenstates $|\Phi_l'\rangle(l = 1, 2)$ of N' and M_i in equations (30) and (31), we have

$$\left(\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}\right)^2 \hat{O}_e|\Phi_l'\rangle = 4[\alpha\Delta(\{n_i + 1\}) - \beta\chi]\Bigg[\Delta(\{n_i + 1\})(Q^\dagger + Q) \\ - \chi f^2(\{n_i + 1\})\prod_{j=1}^{m}(n_j + 1)\sigma_z\Bigg]|\Phi_l'\rangle, \tag{49}$$

which is in a form like equation (45). From equations (47) and (50), we obtain

$$\alpha = -\frac{\Delta(\{n_i + 1\})}{\chi f^2(\{n_i + 1\}) \prod_{j=1}^{m}(n_j + 1)}\beta. \tag{50}$$

Thus in the Hilbert space spanned by the eigenstates $|\Phi_l'\rangle$, we may determine the expression of $\hat{O}_e$,

$$\hat{O}_e = -\frac{\Delta(\{n_i + 1\})}{\chi f^2(\{n_i + 1\}) \prod_{j=1}^{m}(n_j + 1)}\beta(Q^\dagger + Q) + \beta\sigma_z, \tag{51}$$

which is called a pseudo-invariant eigen-operator given in equation (16). Substituting equations (49) and (51) into equation (45), we get

$$\lambda = \Delta^2(\{n_i + 1\}) + 4\chi^2 f^2(\{n_i + 1\}) \prod_{j=1}^{m}(n_j + 1), \tag{52}$$

and further obtain the energy-level gap for this system as

$$\sqrt{\lambda} = \sqrt{\Delta^2(\{n_i + 1\}) + 4\chi^2 f^2(\{n_i + 1\}) \prod_{j=1}^{m}(n_j + 1)} = 2E_{m1}(\{n_i\}), \tag{53}$$

which coincides with the eigen-energy of H in equation (36).

From the above discussion, in order to get the PIEOs, the key point is to find the conservative quantities of the corresponding model.

4. Conclusions

In summary, as described by the supersymmetric generators, we have diagonalized the Hamiltonian of the non-degenerate multimode multi-photon Jaynes–Cummings (JC) model, including any forms of intensity-dependent coupling, field-dependent detuning and field nonlinearity, and obtain its energy eigenvalues and eigenstates. In addition, based on the theory of PIEO, we have also found its pseudo-invariant eigen-operator and directly derived the energy-level gap. It is shown that our methods obtain the energy-level gap formula more directly and clearly than the usual Schrödinger approach. In this sense, we may say that supersymmetry is a natural language to depict the general JC model. These general results and methods adapt to the given mode and any specific forms of intensity-dependent coupling, field-dependent detuning and field nonlinearity. For instance, as $m = 2$, $R = 0$, $G = 1$ and $f = \sqrt{a_1^\dagger a_1 a_2^\dagger a_2}$, its energy-level gap is

$$\Delta E' = 2\sqrt{(\omega_0 - \omega_1 - \omega_2)^2 + 4[\chi(n_1 + 1)(n_2 + 1)]^2},$$

which is the model used by Gao *et al* [16]. For the non-degenerate TTJC model with a Kerr-like medium in [17], i.e. $f = G = 1$, $R = \frac{1}{4}\varepsilon_1\left(a_1^\dagger a_1\right)^2 + \frac{1}{4}\varepsilon_2\left(a_2^\dagger a_2\right)^2 + \varepsilon a_1^\dagger a_1 a_2^\dagger a_2$, the result becomes

$$\Delta E'' = 2\sqrt{\Delta^2(n_1 + 1, n_2 + 1) + 4\chi^2(n_1 + 1)(n_2 + 1)},$$

where $\Delta(n_1 + 1, n_2 + 1) = \omega_0 - \varepsilon - \sum_{j=1}^{2}\left[\left(\frac{\varepsilon_j}{2} + \varepsilon\right)n_j + \omega_j + \frac{\varepsilon_j}{4}\right]$. We believe that these general results will further enrich the investigations of the nonlinear dynamical and statistical properties of this kind of system.

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